ANAPHORA IN A FOUR-VALUED SETTING

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1 Introduction

There is a close connection between presupposition projection and anaphoric accessibility. Both exhibit striking asymmetries in many environments (Karttunen, 1973, Heim, 1982), but the exception that proves the rule is arguably their parallel *symmetric* behavior in disjunctive sentences. As hinted at by Karttunen (1973:p. 180, fn11), disjunctive sentences arguably[1] exhibit symmetric filtering. This is illustrated by the pair in (1). The anaphoric parallel, noticed independently by Evans (1977) and Barbara Partee, is provided in (2).

(1) a. Either Benjamin never smoked, or he continues to do so.
    b. Either Benjamin continues to smoke, or he never did so.

(2) a. Either there isn't any^{x} bathroom, or it_{x}'s in a funny place.
    b. Either it_{x}'s in a funny place, or there isn't any^{x} bathroom.

This close connection suggests a uniform explanation. In this short paper, we build on recent work (particularly that of Rothschild 2017 and Heim 2024) and sketch a static treatment of anaphora which captures the parallelism observed in (1) and (2) via an extension of the trivalent account of presupposition projection. In common with many existing theories of anaphora, the core intuition behind our account is that pronouns are variables (pace Chatain 2025), and thereby introduce *assignment-dependent* presuppositions. As such, a co-indexed indefinite may filter such presuppositions (Rothschild, 2017, Heim, 2024).

In order to characterize the behavior of indefinites, which we treat as projective but not presuppositional, we ultimately move beyond trivalence to a *quadrivalent* system. The idea in a nutshell is that indefinites and pronouns are both projective *but in different ways*. The parallels between anaphoric accessibility and presupposition projection fall out simply because pronouns are presuppositional, that is, their definedness-conditions (unlike that of indefinites) are of the same sort as regular presuppositions. Our account necessitates certain assumptions concerning how the

[1]Recently, Kalomoiros and Schwarz (2024) have provided experimental support for this perspective, contra Hirsch and Hackl (2014).

projective content associated with indefinites interacts with logical connectives—in the final part of this paper, we aim to address this explanatory gap by sketching a quadrivalent extension of the Kleene system developed by George (2008a,b).

2 The background

The trivalent approach to presupposition projection (see, e.g., Peters 1979, Beaver and Krahmer 2001) introduces a third truth-value, #, to stand in for presupposition failure. On this approach, the (semantic) *presupposition* of a sentence ϕ can be defined as the worlds in which it has a bivalent truth-value. In the trivalent tradition, solving the projection problem for presuppositions amounts to positing trivalent counterparts for bivalent truth-functional operators. The universally agreed-upon trivalent semantics for negation maps a # argument to #. This of course predicts that a sentence of the form "not ϕ" inherits the presuppositions of ϕ. Stating an adequate trivalent semantics for the 2-place truth-functional operators is somewhat more involved, given the substantially larger possibility space.[2] Taking conjunction as a starting point, it is generally agreed that Peters's (1979) so-called *Middle Kleene* semantics adequately describes how presuppositions project in conjunctive sentences (see also George 2008a,b, as well as Spector to appear for a recent overview).

$\phi \wedge^{\text{mk}} \psi$	1	0	#
1	1	0	#
0	0	0	0
#	#	#	#

Figure 1: Middle Kleene conjunction

Anaphoric accessibility tracks presupposition projection—in other words, an indefinite antecedent in an initial conjunct is 'accessible' to a pronoun in a subsequent conjunct, but not vice versa. Such asymmetries are the bread and butter of *dynamic* approaches to anaphoric dependencies (Heim, 1982, Kamp, 1981, Groenendijk and Stokhof, 1991). Instead, our plan is to show one way to extent the *static*, trivalent account of presupposition projection to ground the asymmetric availability of discourse anaphora.

3 The proposal

3.1 Pronouns and assignments

We adopt the minimal assumption that (i) pronouns are variables, and (ii) assignments are partial. Thus e.g. "it_x is upstairs" has an assignment-dependent presupposition. We therefore generalize the notion of semantic presupposition of a sentence ϕ: we take it to be those *world-assignment pairs* in which ϕ has a bivalent truth-value. (3) has a bivalent truth-value at any (w,g) so long as $x \in \textbf{dom}(g)$, hence (3) *presupposes* that x is defined.

[2] Given an n-place bivalent truth-functional operator, there are $3^{3^n - 2^n}$ possible trivalent extensions.

(3) $$[\![\text{It}_x \text{ is upstairs}]\!]^{w,g} = \begin{cases} 1 & x \in \mathbf{dom}(g) \,\&\, g(x) \text{ is upstairs in } w \\ 0 & x \in \mathbf{dom}(g) \,\&\, g(x) \text{ is not upstairs in } w \\ \# & \text{otherwise, i.e. } x \notin \mathbf{dom}(g) \end{cases}$$

3.2 Indefinites and assignments

If "it$_x$ is upstairs" presupposes that x is defined then "there is a^x bathroom" should also entail that this is so (Rothschild, 2017, Heim, 2024). In other words, we can start with the following truth-clause for a simple sentence involving an indefinite:

(4) $[\![\text{There is a}^x \text{ bathroom}]\!]^{w,g} = 1$ if $x \in \mathbf{dom}(g)$ and $g(x)$ is a bathroom in w

Since we are in a trivalent setting, we now need to decide on the conditions under which "there is a^x bathroom" denotes 0 vs #. Let us distinguish between two types of world-assignment pairs at which (4) is not true.

(5) a. **Case (i)** There is no bathroom in w
 b. **Case (ii)** There is a bathroom in w but either $x \notin \mathbf{dom}(g)$ or $g(x)$ isn't a bathroom in w

Evidently, "there is a^x bathroom" should be 0 in case (i). The status of case (ii) is less clear however. Let us consider two options. On the *assertive* analysis, "there is a^x bathroom" is 0 in both case (i) and case (ii), i.e., "there is a^x bathroom" asserts that x is defined (and x is a bathroom). On the *presuppositional* analysis, "there is a^x bathroom" is 0 in case (i) but is # in case (ii), i.e., "there is a^x bathroom" *presupposes* that: *if there is a bathroom, x is a bathroom*. The assertive analysis is due to Rothschild (2017) and Heim (2024) while the presuppositional analysis is due to Spector (forthcoming) and, technicalities aside, Mandelkern (2022). We'd like to argue that both options are, in a sense, correct.

The presuppositional analysis has an upper hand in its treatment of negated indefinites. After all, since "not" maps # to #, the presuppositional analysis predicts by design that "there isn't a^x bathroom" is 1 in (w,g) iff there is no bathroom in w, regardless of g. The assertive analysis, on the other hand, seems far too weak. It predicts that "there isn't a^x bathroom" is 1 in (w,g) if x is not in the domain of g, regardless of w. To see what's wrong with this result, suppose we say that ϕ is true in w iff there is a g such that $[\![\phi]\!]^{w,g} = 1$.[3] If so then on the assertive analysis "there isn't a^x bathroom" is predicted to be a necessary truth despite the fact that "there is a^x bathroom" is predicted to be contingent!

To appreciate the advantage of the assertive analysis, let us observe that any pragmatic framework that's taken to complement our semantics must at the very least be able to distinguish between variables that are introduced by indefinites and variables that are introduced by pronouns that lack an indefinite antecedent. Given the rest of our assumptions, this is not trivial. E.g. observe that "it$_x$ is upstairs" and "there is a^x thing upstairs" are predicted to be 1 in the exact same world-assignment pairs. Since the use-conditions of these sentences are obviously different, the pragmatics must be able to distinguish between them this. Now, on the assertive analysis there is a very elegant way of doing this based on "domain sensitivity" (what Rothschild 2017 and Heim 2024 call 'definedness sensitivity").

[3]We will define truth below in a slightly different way, but our definition doesn't solve the problem for the assertive analysis.

(6) *Domain sensitivity:*
x is domain sensitive in ϕ, $x \in DS(\phi)$, iff there are w, g, g' such that g and g' are identical except that $x \notin \mathbf{dom}(g')$ and $[\![\phi]\!]^{w,g} \neq \#$ while $[\![\phi]\!]^{w,g'} = \#$.

For example, in (7), x is domain sensitive in (a) but it is *not* domain sensitive in (b) or (c). The latter crucially relies on the Middle Kleene treatment of conjunction which guarantees that if the left conjunct is false, the whole conjunction is false regardless of the second conjunct.

(7) a. It_x is upstairs.
b. There's a^x bathroom.
c. There's a^x bathroom, and it_x's upstairs.

As useful as domain sensitivity is (see below), it is heavily dependent on the assertive treatment of indefinites. We invite the reader to verify that, given (6) and the presuppositional analysis, x is domain sensitive in all three of the sentences in (7).[4]

So we are faced with a dilemma. On the one hand, the assertive analysis of indefinites allows us to rely domain sensitivity to tease pronouns apart from indefinites but leads to inadequate results for negated indefinites. On the other hand, the presuppositional analysis of indefinites makes decent predictions for negated indefinites but does not allow us to distinguished between pronouns and indefinites via domain sensitivity. What to do?

We propose to have our cake and eat it too. The assertive analysis is correct that (7b) never denotes #, but the presuppositional analysis is also correct that (7b) sometimes denotes neither 1 nor 0. Specifically, we propose to switch to a quadrivalent semantics based on *four* truth-values $\{1, 0, \#, \star\}$. We now have the following option.

(8) $$[\![\text{There's a}^x \text{ bathroom}]\!]^{w,g} = \begin{cases} 1 & x \in \mathbf{dom}(g) \,\&\, g(x) \text{ is a bathroom in } w \\ 0 & \text{there is no bathroom in } w \\ \star & \text{otherwise (i.e., case (ii) in (5))} \end{cases}$$

Assuming that "not" maps # to # and $\star$ to $\star$, (8) predicts, like the presuppositional analysis, that "there isn't a^x bathroom" is 1 iff there is no bathroom. Like the assertive analysis, on the other hand, (8) predicts that x is domain sensitive in (7a) and not in (7b). The latter is because our definition of domain sensitivity in (6) cares only about # and does not distinguish between 1, 0 and $\star$. With our dilemma now resolved, let us now turn to broader semantic and pragmatic issues that immediately arise on this new perspective.

[4] One alternative possibility we might consider, on the presuppositional analysis, is to define a more sophisticated notion of domain sensitivity that can distinguish between the conditional presupposition of "there is a^x bathroom" and the unconditional presupposition of "it_x is upstairs". This won't obviously help since, given standard treatments of presupposition projection, pronouns in embedded contexts can give rise to parallel conditional presuppositions (we're grateful to an anonymous reviewer for raising this point):

(i) Mary has a^x sibling.
presupposes: *If Mary has a sibling, then x a sibling of Mary*

(ii) Either Mary is an only child, or she_x is Mary's sibling.
presupposes: *If Mary has a sibling, then x is a sibling of Mary*

Currently, the semantics is set up in such a way that a sentence such as "there's a^x bathroom" is $\star$ at some world-assignment pairs where it would be classically true, i.e., any (w,g) s.t., there's a bathroom in w, but $g(x)$ is either undefined or not a bathroom in w. We can define a notion of *truth* relative to (w,g), that reinstates classical truth-conditions by existentially quantifying over assignments that *agree* with g on the domain sensitive variables, (9a). Since there are no domain sensitive variables in "there's a^x bathroom", *truth* at g just amounts to the requirement that there is some assignment at which it is 1. Of course, a pre-condition for *truth* at (w,g) is that g in fact provides values for the domain sensitive variables—this is encoded as the *domain condition* in (9b).[5]

(9) a. ϕ is **true** at (w,g) if $\exists h, \forall x \in DS(\phi), h(x) = g(x), \& [\![\phi]\!]^{w,h} = 1$
b. ϕ is **defined** at (w,g) if $DS(\phi) \subseteq \mathbf{dom}(g)$ (the *domain condition*)

This is sufficient to account for conjunctive discourse anaphora as in (10) since there are no domain sensitive variables in (10)—we'll go through this carefully in the following section. (11) can also be accommodated if we assume that discourse sequencing is interpreted as conjunction (Heim, 1982).

(10) There's a^x bathroom and it_x's upstairs.

(11) There's a^x bathroom. It_x's upstairs.

Perhaps a more interesting possibility is to account for *bona fide* discourse anaphora as in (11) by invoking a Heim-Stalnaker notion of the discourse context as a *set of world-assignment pairs*, together with a notion of how asserting a sentence ϕ at a context c results in an *updated* context $c[\phi]$ (see Heim's notion of a *file*). For instance, we could consider an (anaphorically) initial context to be one in which we take the product of a set of possible worlds with the set of all partial assignments (provided a finite set of variables). Heimian familiarity can be cashed out as the requirement that ϕ satisfies the domain condition at *every* $(w,g) \in c$, (12a). Assertion simply amounts to the usual Stalnakerian notion—world-assignment pairs at which the sentence is not 1 are eliminated. Note crucially that asserting "There is a^x bathroom" eliminates not just (w,g)s at which it is 0, but also (w,g)s at which it is $\star$. The resulting context is guaranteed to be one at which x is familiar, since (w,g)s where g does not map x to a bathroom in w will have been removed.[6]

(12) Given a sentence ϕ, and a context c:
a. $c[\phi]$, is *defined* only if $\forall (w,g) \in c$, $DS(\phi) \subseteq \mathbf{dom}(g)$ (*familiarity*).
b. If defined, $c[\phi] = \{ (w,g) \in c \mid [\![\phi]\!]^{w,g} = 1 \}$

We close this section by noting that our proposal needs to be supplemented with a condition that prevents indefinites from simply being interpreted as variables. Following Heim (1982), we call this the *novelty condition*, and it is defined in (13). "There's a^x bathroom" will violate *novelty* at any (w,g) where $x \in \mathbf{dom}(g)$, since "There's it_x" would not violate the domain condition.[7]

[5] As usual, a sentence can be considered *false* if it is *defined* and not *true*.

[6] The pragmatic proposal we've sketched here is clearly eliminative, in the sense that updating a context set always results in a smaller context set (if defined) (see Rothschild and Yalcin 2016, 2017). We believe that our semantic proposal can also be supplemented with a *non-eliminative* pragmatic component, by taking (minimal) extensions of (w,g)s at which the sentence is $\star$. We leave a more thorough consideration of the pragmatic possibility space to future work.

[7] Whether competition with ungrammatical sentences makes any sense is a question that we put aside here.

(13) ϕ satisfies the **novelty condition** at (w,g) only if there is no indefinite NP in ϕ which can be replaced with a co-indexed pronoun without violating the domain condition.

3.3 Conjunction

We must now say something about how $\star$ projects. A very natural assumption, call it Undefinedness Uniformity (UU), is that $\star$ projects just like #, i.e. according to the Middle Kleene schema for "and". The result is illustrated below. We'll return to how this truth-table is to be derived on principled grounds later.

$\phi \wedge^{mk*} \psi$	1	0	#	$\star$
1	1	0	#	$\star$
0	0	0	0	0
#	#	#	#	#
$\star$	$\star$	$\star$	$\star$	$\star$

Let's consider what this analysis predicts for a sentence like (14).

(14) There's a^{x} bathroom, and it$_{x}$'s upstairs.

The denotation of this sentence relative to (w,g) is given in (15). Note that the first conjunct is never #, and if the first conjunct is 1, the second conjunct can never be #, since the truth of the first conjunct guarantees that $x \in \textbf{dom}(g)$. So the sentence can never be #, reflecting the fact that the pronoun's presupposition is filtered by its indefinite antecedent. We leave it to the reader to verify that we predict (14) to be true in w iff there is a bathroom upstairs in w. A sentence like "There's a^{x} bathroom, and a^{x} bathroom's upstairs" is ruled out by novelty since the second "a^{x} bathroom" can be replaced with "it$_{x}$", resulting in (14), without violating the domain condition in any context, since x is not domain sensitive in the (14) to begin with.

(15) $$\begin{cases} 1 & x \in \textbf{dom}(g) \,\&\, g(x) \text{ is a bathroom upstairs in } w \\ 0 & \text{there's no bathroom in } w \text{ or, } x \in \textbf{dom}(g) \,\&\, g(x) \text{ is a bathroom that's not upstairs in } w \\ \star & \text{there is a bathroom in } w \text{ and either } x \notin \textbf{dom}(g) \text{ or } g(x) \text{ is not a bathroom in } w \end{cases}$$

We (also[8]) leave it to the reader to verify that, due to the asymmetrical nature of Middle Kleene, x is domain sensitive in (16) and consequently this sentence can never satisfy both the domain condition and the novelty condition simultaneously. The well-known left-to-right nature of anaphora in conjunction is, therefore, accounted for.

(16) It$_{x}$'s in a funny place, and there's a^{x} bathroom.

4 Anaphora and cataphora in bathroom sentences

Recall from §1 that anaphoric accessibility in disjunction, much like presupposition projection, is symmetric. This immediately entails that we cannot analyse disjunction on the assumption that it

[8] Sorry. Space limitations.

projects both # and $\star$ according to the Middle Kleene schema, since the latter will automatically encode a left-to-right asymmetry much like the case of conjunction that we just saw. If we maintain our commitment to UU, then, we need to apply a schema to both # and $\star$ that yields a *symmetric* truth-table. There are two familiar options here, neither of which work.

The so-called Weak Kleene schema says that a disjunction denotes # (or $\star$) as soon as one of the disjuncts denotes # (resp. $\star$). But this is clearly inadequate. It means that in (17) the variable x is domain sensitive since the right disjunct is # whenever x is not in the domain of the assignment. This in turn means that (17) can never satisfy the domain and novelty conditions simultaneously.[9]

(17) There isn't a^{x} bathroom or it_{x} is upstairs

The other option is to use the so-called Strong Kleene schema. The hallmark of the latter, when applied to disjunction, is that the whole disjunction comes out 1 as soon as one of the disjuncts is 1 *regardless of the value of the other disjunct*. But this is clearly inadequate as well. It means that (17) denotes 1 relative to (w,g) as soon as $g(x)$ is upstairs in w regardless of whether $g(x)$ is a bathroom in w or not! In other words, a uniform treatment of the two undefined values using the Strong Kleene schema fails to establish an anaphoric link between "a^{x} bathroom" and "it_{x}".

For better or worse, there are over sixteen million quadrivalent extensions of classical disjunction, and it appears that we need to explore other options. As it happens one of these extensions works quite nicely from our perspective though it happens to violate UU—that is, it treats # and $\star$ differently. Interestingly enough, though, this extension treats each of # and $\star$ along one of the familiar schemas. Beginning with bivalent disjunction, we first add # using the Strong Kleene schema and then we take the result and add $\star$ using the Weak Kleene schema (see §5 for a take on how to derive this truth-table on more principled grounds).

$\phi \vee^{sk*} \psi$	1	0	#	$\star$
1	1	1	1	$\star$
0	1	0	#	$\star$
#	1	#	#	$\star$
$\star$	$\star$	$\star$	$\star$	$\star$

To see how this quadrivalent extension of classical disjunction accounts for anaphora across disjuncts, consider the sentence in (17) again and keep in mind that since our disjunction is symmetric, the order of disjuncts is irrelevant.

To begin with, we observe that the left disjunct can't be # and the right disjunct can't be $\star$. So right off the bat we can ignore the third row and the fourth column of disjunction's truth-table. Next we observe that if the first disjunct is 0 then "there is a^{x} bathroom" is 1, which means that x is defined. Therefore the disjunction as a whole can never be #, reflecting the fact that the pronoun's presupposition is successfully filtered. Next we observe that the sentence is predicted to be 0 iff both disjuncts are 0, i.e. $g(x)$ is a bathroom but it is not upstairs. Putting all the pieces together, then, we have the following analysis for (17)'s denotation in (w,g).

[9] The Middle Kleene schema, even ignoring the problem of symmetry, has the same problem if the order of disjuncts in (17) is flipped.

(18) $$\begin{cases} 1 & \text{there is no bathroom in } w \text{ or, } g(x) \text{ is a bathroom upstairs in } w \\ 0 & g(x) \text{ is a bathroom in } w \text{ but it is not upstairs in } w \\ \star & \text{there is a bathroom in } w \text{ and either } x \notin \mathbf{dom}(g) \text{ or } g(x) \text{ is not a bathroom in } w \end{cases}$$

Applying our definition of truth, we predict that (17) is true iff either there is no bathroom or at least one bathroom is upstairs. Note that the predicted truth conditions are *existential*. Elliott (2024) argues at length (contra, e.g., Krahmer and Muskens 1995) that this is a reasonable prediction. To briefly motivate this reading, consider (19) from Elliott (2024) and its cataphoric variant in (20). Intuitively, both sentences are judged true, even if Gabe has multiple umbrellas, so long as he remembered to bring at least one.

(19) Either Gabe doesn't have an^{x} umbrella, or he remembered to bring it$_{x}$.

(20) Either Gabe remembered to bring it$_{x}$, or he doesn't have an^{x} umbrella.

Finally let us simply flag here that our account of novelty predicts, correctly, that both sentences below, with co-indexed indefinites, should be unacceptable since in both cases the unnegated indefinite can be replaced with a co-indexed pronoun without violating the domain condition.

(21) Either Gabe doesn't have an^{x} umbrella, or he remembered to bring something^{x}.

(22) Either Gabe remembered to bring something^{x}, or he doesn't have an^{x} umbrella.

It's perhaps worthwhile to emphasize that cases like (20) are seemingly incompatible with a syntactic novelty condition based on leftness (Heim, 1982:Chapter 2), since in order to make sense of the truth conditions of (20) it is necessary to assume that the indefinite under negation is conindexed with the preceding pronoun.[10]

5 Deriving projection

A four-valued approach to presupposition and anaphora seemingly exacerbates the problem of formulating an *explanatory*, rather than merely descriptive, account of presupposition projection and anaphoric accessibility. What is minimally necessary in order to make the narrative we have sketched here compelling is an algorithm which allows us to derive, in a principled fashion, the quadrivalent truth-tables we have posited to be descriptively adequate.

We have exploited the assumption that # projects in conjunctive sentences according to the Middle Kleene (asymmetric) schema, and in disjunctive sentences according to the Strong Kleene (symmetric) schema. This seems to be necessary in order to account for projection facts, independently of anaphora (see especially Kalomoiros and Schwarz 2024). George (2008a) proposes an algorithm which derives this Kleene system. George's (2008a) idea, informally, is that

[10] Spector (forthcoming) proposes a syntactic novelty condition which is compatible with cataphoric bathroom sentences while still ruling out (21) and (22), by simply ruling out any co-indexing of indefinite NPs by fiat. This may turn out to be too strong however, if Groenendijk and Stokhof (1991) are right in suggesting that cases like (i) involve co-indexed indefinites:

(i) A professor or an assistant professor will attend the meeting of the university board.
He will report to the faculty. (Groenendijk and Stokhof, 1991:p. 88)

arguments to a connective are evaluated *incrementally*. If at any point in incremental evaluation, a #-argument is responsible for ruling out any hope for truth (in a Strong Kleene sense), then # is 'disappointing' and it projects, otherwise incremental evaluation continues.

Here we will tentatively sketch an extension of George's algorithm to a four-valued setting, focusing on 2-place logical connectives, with the hope that the extension to the more general case is easy to reconstruct. The core intuition behind our extension is that $\star$ is a kind of undefinedness—as a result, as soon as $\star$ is encountered incrementally, it projects, in line with so called 'Weak Kleene' systems. We also add to George's procedure the following assumption of *minimal evaluation*:

(23) **Minimal evaluation:** if, at any point during incremental evaluation of a truth-functional operator, *falsity* is guaranteed (in a Strong Kleene sense), then return 0 without evaluating further.

We'll describe informally how this achieves the desired results, deferring a more fully-fledged formal treatment. Starting with the logical connective **and**: if the first argument encountered is 1, the value of the sentence depends on the second argument—both # and $\star$ project in this case; # because it is disappointing and $\star$ due to undefinedness. If the first argument encountered is 0, then by *minimal evaluation*, return 0 without evaluating further. This is because, regardless of whether the next argument is $1, 0$ or #, the statement will be 0. If the first argument is #, therefore # is disappointing and it projects. If the first argument is $\star$, $\star$ projects. This derives the extended Middle Kleene truth-table for conjunction.

Turning to **or**, if the first argument encountered is 1, then the second argument is evaluated—anything except $\star$ results in 1, but $\star$ nevertheless projects. If the first argument encountered is 0, then the second argument is evaluated, and both # and $\star$ project. If the first argument is #, because as before # is not disappointing. If the first argument is $\star$, it projects. *Minimal evaluation* doesn't have any effect in disjunctive sentences, since it can't be determined whether or not a disjunctive sentence is false before encountering the second argument. The interplay of these principles derives the extended Strong Kleene truth-table for disjunction.

This is a rather impressionistic sketch, but hopefully it suffices to show, as a proof of concept, how to develop an *explanatory* account of presupposition and anaphora in a four-valued setting.

6 Conclusion and open issues

One major challenge for the future is an explanatory, trivalent theory of projection in conditional statements, grounded in an independently motivated semantics for conditionals. Along similar lines, it is not at all trivial to extend our account to donkey anaphora in quantificational sentences. It remains to be seen whether the trivalent account of projection in quantificational sentences (which comes with its own set of complications; see Fox 2013) delivers reasonable results in tandem with the four valued approach. Zooming out, there is significant work to be done in developing an understanding of how the four-valued approach fairs relative to alternative, explanatory theories of anaphora that have emerged in recent years. Our account has certain parallels with Mandelkern's (2022) static *bounded* theory, which as Mandelkern (p.c.) notes can be understood as a four-valued system, where sentences are evaluated relative to two dimensions, which have two values each. There is also a parallel with Elliott's (2020) trivalent dynamic account of anaphora, which can be presented as a four-valued relational semantics—on Elliott's account, a sentence, relative to a pair of assignments, may be true, false, a presupposition failure, or undefined. Mandelkern's and

Elliott's theories are founded on quite different assumptions both from each other and from the present theory, but the fact that they are implicitly four-valued suggests deeper parallels.

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